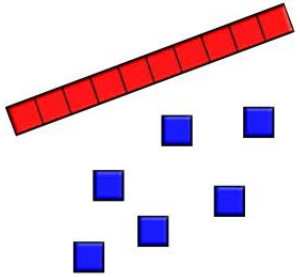
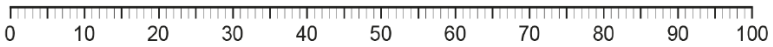
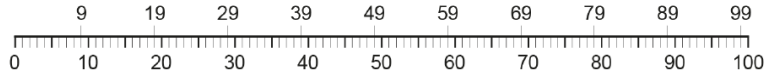
Hopton's Representations Guidance

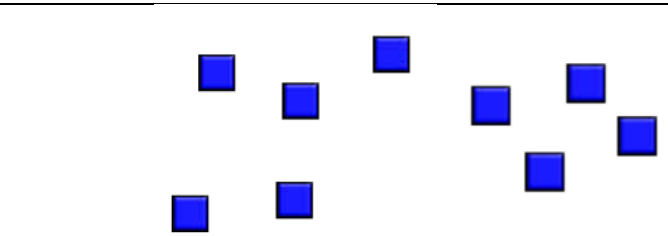
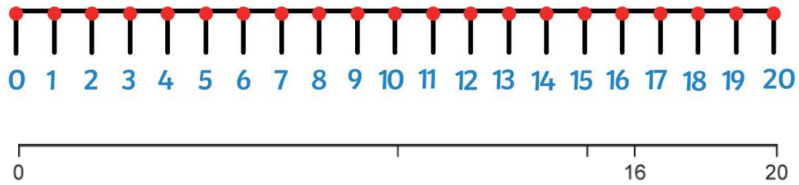
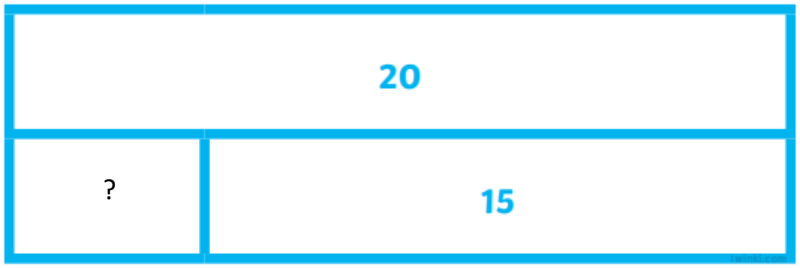
March 2021

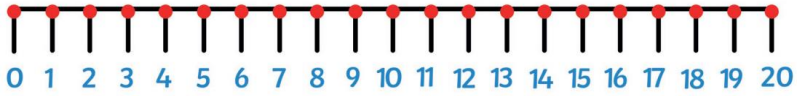
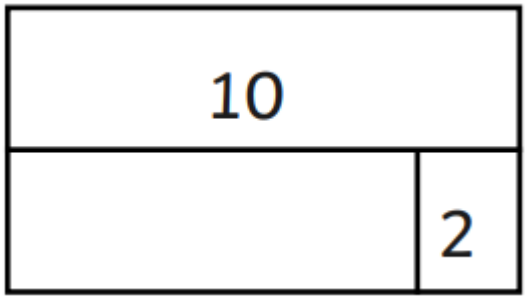
This document is designed to be used alongside our Hopton Maths Calculations policy. It provides a guide for teachers to what manipulatives and pictorial representations to use to highlight the mathematical structure being taught in each 'ready to progress' criteria. We have focused on the 'ready to progress' criteria (DfE June 2020) as these are the core mathematical areas that children must understand before progressing onto the next year. To see the full coverage of learning objectives for each year see the Hopton progression map.

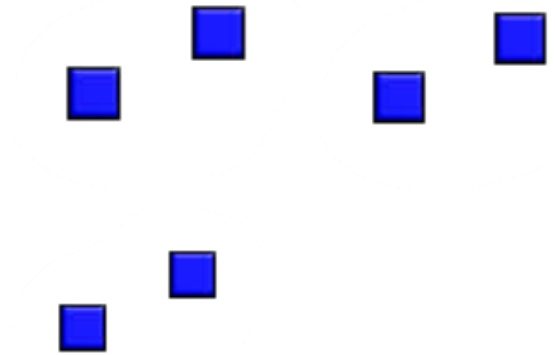
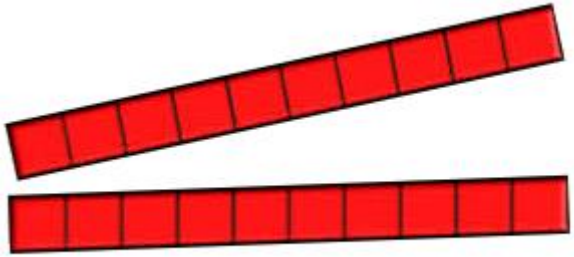
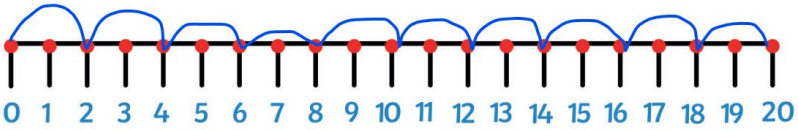
In the table there are two columns – the first shows the core manipulatives/pictorial representations that the children should use and be familiar with. The second column shows complementary manipulatives/pictorial representations that the teacher may choose to use when using variation

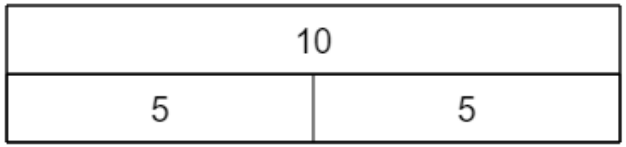
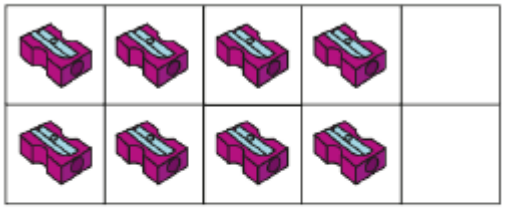
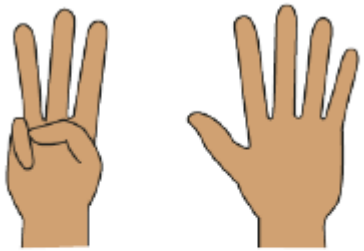
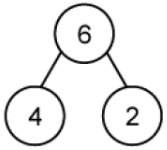
Year 1

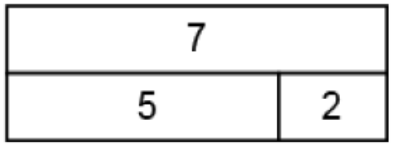
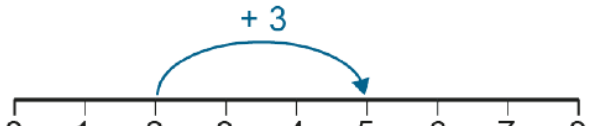
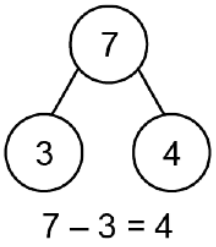
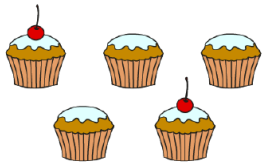
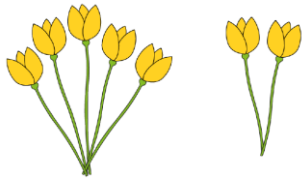
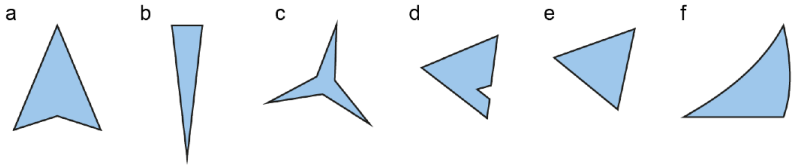
	Core Concrete > pictorial > abstract	Complementary Contexts
Count within 100, forwards and backwards, starting with any number.	   1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100 1,000 2,000 3,000 4,000 5,000 6,000 7,000 8,000 9,000 100 200 300 400 500 600 700 800 900 10 20 30 40 50 60 70 80 90 1 2 3 4 5 6 7 8 9	  1 2 3 4 5 6 7 8 9 10 20 30 40 50 60 70 80 90 100 200 300 400 500 600 700 800 900

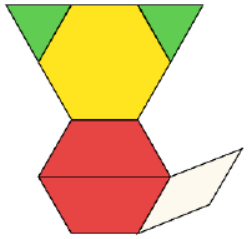
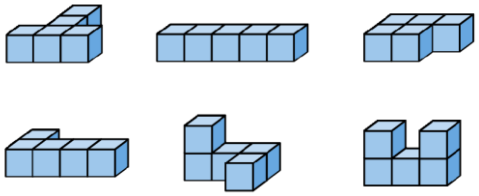
	Without support Count on and back to 20, to 50, to 100. Begin at different numbers.	
Reason about the location of numbers to 20 within the linear number system, including comparing using < > and =	  Fill in the missing numbers on a blank number line and justify their reasons.  Find the missing amounts in Bar Models justifying their reasons. Without support Answer questions such as: Tell me a number less than 5.	

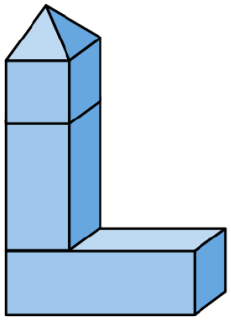
	Tell me a number more than 17. What number is 1 less than 16? What number is 1 more than 12?	
Develop fluency in addition and subtraction facts within 10.	 7 + 3 = 10   Find the missing numbers on a Bar Model. Without Support Answer questions such as: 4 + 3 = 5 + 2 = 6 + ? = 8 ? + 3 = 9 I cycled 4km to get to my friend's house, and then cycled another 3km with my friend. How far have I cycled?	

Count forwards and backwards in multiples of 2, 5 and 10, up to 10 multiples, beginning with any multiple, and count forwards and backwards through the odd numbers.	 Use Diennes to create groups of 2 to count on.  Use Diennes rods to count on in 10s  Without support	

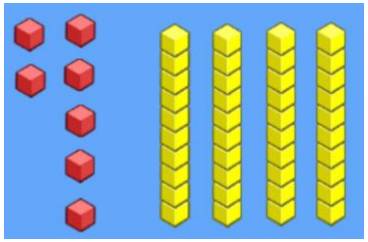
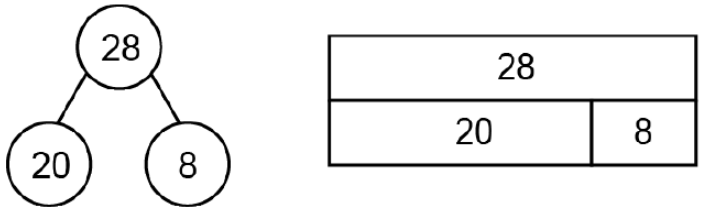
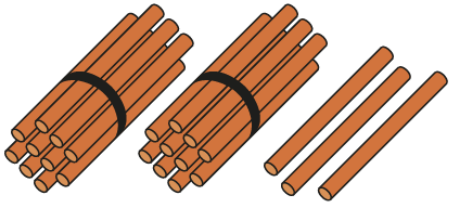
Compose numbers to 10 from 2 parts, and partition numbers to 10 into parts, including recognising odd and even numbers.	Use Diennes to partition numbers:  4 + 3 = 7 Bar Model  Recognise even numbers as being made up of 2s:  = 6  = 10 Recognise odd numbers as being made up of 2s and 'one more':  = 5  = 9 Without support Group numbers based on whether they are even or odd. Answer questions such as:	 **Figure 18: 8 represented as 2 rows of 4**  **Figure 15: 8 represented as 3 fingers and 5 fingers** Use cherry model to partition: 

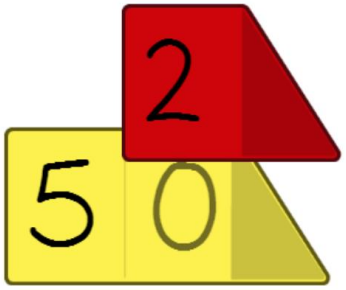
	Mother duck is in the water with her 6 ducklings. There are 2 ponds. How many ducklings could be in each pond?	
Read, write and interpret equations containing addition (+), subtraction (-) and equals (=) symbols, and relate additive expressions and equations to real-life contexts.	Bar model This one is showing subtraction:  7 – 2 = 5 Number Line  2 + 3 = 5 Without support Write four addition and subtaction number family facts using equipment and then write them down. For example, using 7 Diennes cubes write 7 = 4 + 3 7 = 3 + 4 4 = 7 – 3 3 = 7 - 4	 7 – 3 = 4 Cherry Model Real life contexts:  
Recognise common 2D and 3D shapes presented in different orientations, and know that rectangles,	Recognise 2D and 3D shapes from plausible distractors:  	

triangles, cuboids and pyramids are not always similar to one another.	Which ones are triangles? Without support Find three squares from a set of 2D shapes including plausible distractors.	
Compose 2D and 3D shapes from smaller shapes to match an example, including manipulating shapes to place them in particular orientations.	Use shapes to make a pattern block picture:  Use tangram pieces to copy pictures:  Make different arrangements from a set number of cubes and use spatial language to compare: 	

	Make compound 3D shapes from examples: 	

Year 2

	Core concrete > pictorial > abstract	Complementary Contexts
Recognise the place value of each digit in two-digit numbers and compose and decompose two-digit numbers using standard and non-standard partitioning.	Use Diennes to represent numbers and make numbers from Diennes illustrations: Make 53 from Diennes.  What number is this? Use place value arrow cards to partition and combine digits:	Bar Model and Cherry Model:  Real life objects such as: 

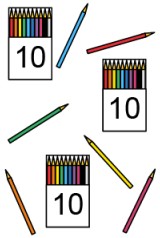


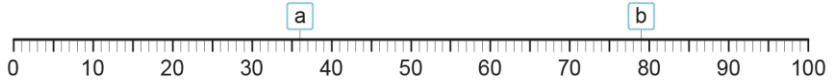
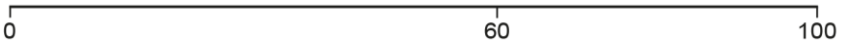
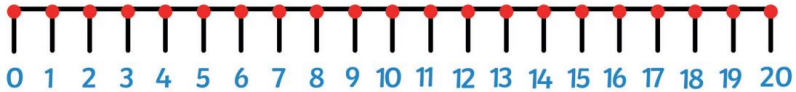
Use coins to represent numbers:

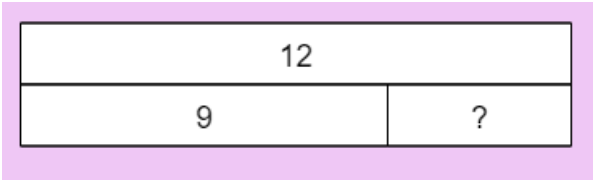
Spot the patterns of digits on a hundred square:

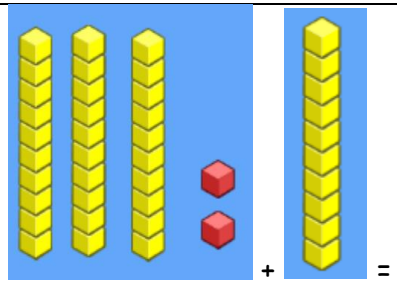
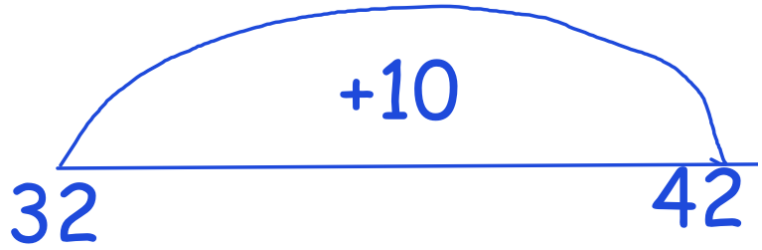
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

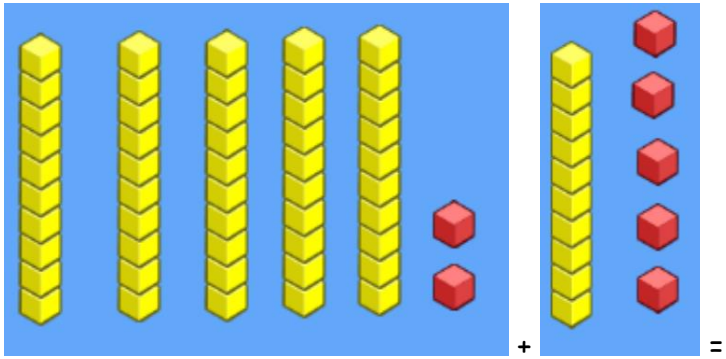
Without support:

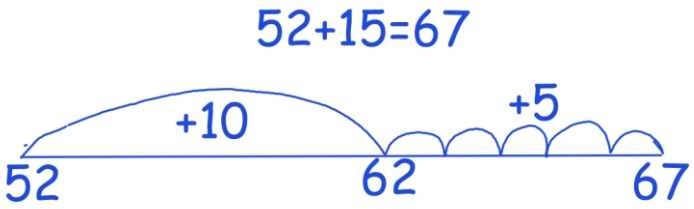
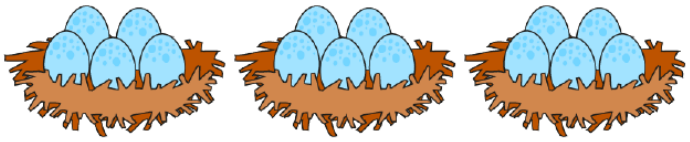


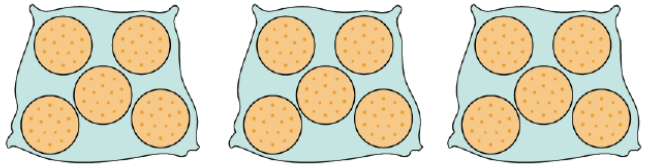
	Partition into a multiple of ten and another two-digit number, in different ways (for example, 68 can be partitioned into 50 and 18, into 40 and 28, and so on) Partition into a two-digit number and a one-digit number, in different ways (for example, 68 can be partitioned into 67 and 1, 66 and 2, and so on)	
Reason about the location of any two-digit number in the linear number system, including identifying the previous and next multiple of 10.	Reason about where numbers are on a number line:   Without support Pupils should be able to say the multiple of 10 above and below a number.	
Secure fluency in addition and subtraction facts within 10, through continued practice. Add and subtract across 10.	Use a number line:  Use a hundred square to count on or back: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100	

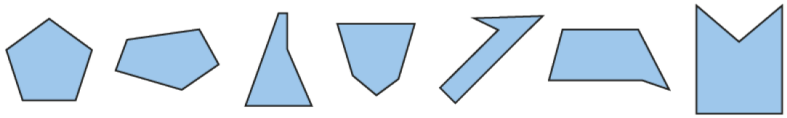
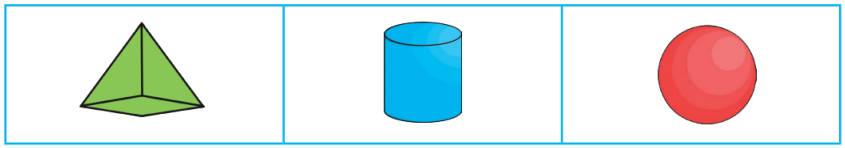
	Without support Pupils should be able to answer questions such as 7+5=12 15-9=6 without using fingers, number line or any other support.	
Recognise the subtraction structure of 'difference' and answer questions of the form, "How many more…?".	Number Line  Bar Model  Without support 19 + □ = 25 What is the difference between Saskia's score and Paul's score? I have £18 and want to buy a game which costs £27. How much more money do I need?	
Add and subtract within 100 by applying related one-digit addition and subtraction facts: add	Use Diennes:	

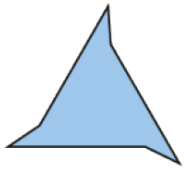
and subtract only ones or only tens to/from a two-digit number.	 + = 32 + 10 = Use a number line that children have drawn themselves to add or subtract:  Move around the hundred square: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100	

	Without support Without number lines etc: Add and subtract two multiples of ten 60+20=80 and 80-20=60 Add and subtract ones to/from a two digit number 52+4=56 and 56–4=52 Add and subtract multiples of ten to/from a two digit number 55-30=25 and 25+30=55 Subtract ones from a multiple of ten 40-2=38	
Add and subtract within 100 by applying related one-digit addition and subtraction facts: add and subtract any 2 two-digit numbers.	Use Diennes:  52 + 15 = Move around the hundred square:	

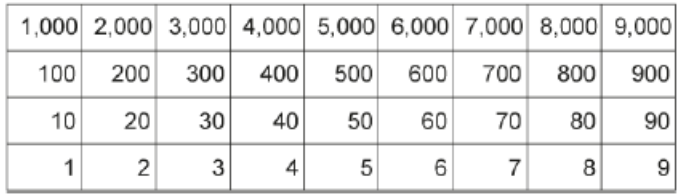
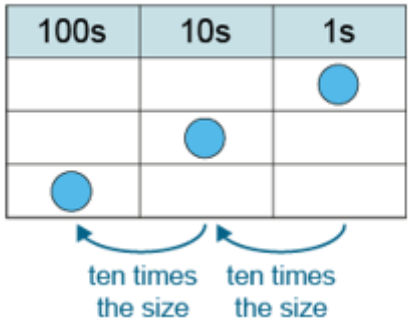
	<table><tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr><tr><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr><tr><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td></tr><tr><td>31</td><td>32</td><td>33</td><td>34</td><td>35</td><td>36</td><td>37</td><td>38</td><td>39</td><td>40</td></tr><tr><td>41</td><td>42</td><td>43</td><td>44</td><td>45</td><td>46</td><td>47</td><td>48</td><td>49</td><td>50</td></tr><tr><td>51</td><td>52</td><td>53</td><td>54</td><td>55</td><td>56</td><td>57</td><td>58</td><td>59</td><td>60</td></tr><tr><td>61</td><td>62</td><td>63</td><td>64</td><td>65</td><td>66</td><td>67</td><td>68</td><td>69</td><td>70</td></tr><tr><td>71</td><td>72</td><td>73</td><td>74</td><td>75</td><td>76</td><td>77</td><td>78</td><td>79</td><td>80</td></tr><tr><td>81</td><td>82</td><td>83</td><td>84</td><td>85</td><td>86</td><td>87</td><td>88</td><td>89</td><td>90</td></tr><tr><td>91</td><td>92</td><td>93</td><td>94</td><td>95</td><td>96</td><td>97</td><td>98</td><td>99</td><td>100</td></tr></table> Use a blank number line that children have drawn themselves to add or subtract:  Without adult support Answer problems such as What is the total cost of the bicycle and construction set? Yasmin wants to buy the construction set. She has saved £15. How much more money does Yasmin need to save?	
Recognise repeated addition contexts, representing them with multiplication equations and calculating the product, within the 2,	Coins  10+10+10=30 	Real life contexts: 

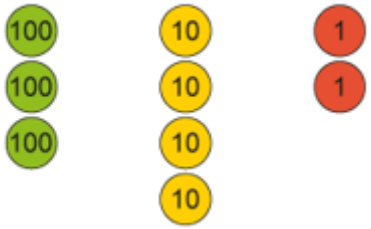
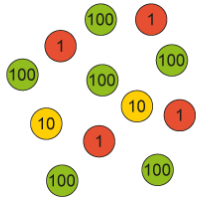
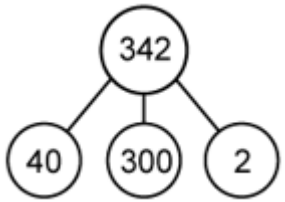
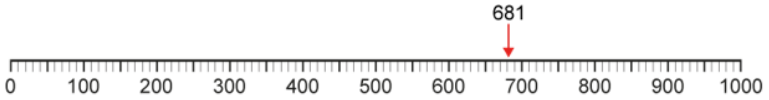
5 and 10 multiplication tables.	2+2+2+2+2=10 Relate repeated addition to 'lots of': 5+5+5=3x5 6x2-2+2+2+2+2+2 and then move to 3x5=15 6x2=12 Without support Complete the calculations: 3x5= 7x10= 8x2= There are 7 year-groups in Winterdale School. Each year-group has 2 classes. How many classes are in the school?	
Relate grouping problems where the number of groups is unknown to multiplication equations with a missing factor, and to division equations (quotitive division) For example □x5=15	Real life examples  □x5=15 Bar Model  How many fives make 15? Without support	

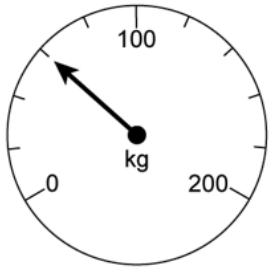
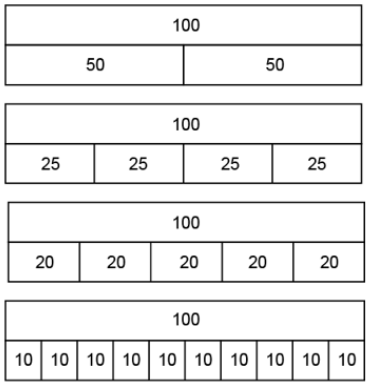
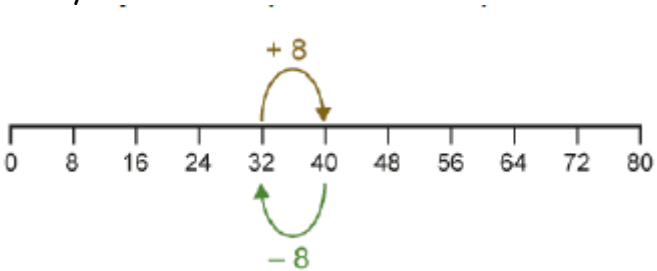
	Miss Robinson asked Harry to get 60 apples from the kitchen. The apples come in bags of 10. How many bags does Harry need to get? Diego has some 5p coins. He has 40p altogether. How many 5p coins does Diego have?	
Use precise language to describe the properties of 2D and 3D shapes, and compare shapes by reasoning about similarities and differences in properties.	Pupils should know what a polygon is. They should understand that polygon names are based on their number of sides/vertices and not a mental picture.  A variety of pentagons. Pupils don't need to know the terms 'parallel' , 'perpendicular' or different types of angles but they should experience seeing shapes with these.  Which shape is the odd one out? Why? Without support How many sides does this shape have? What is the name of this shape?	

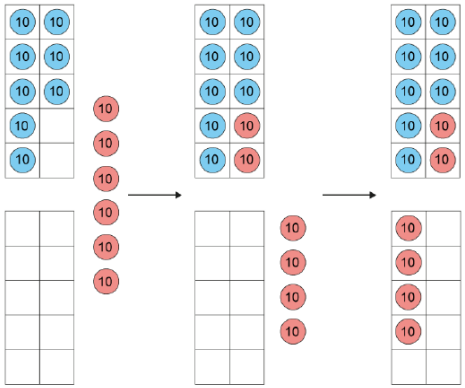


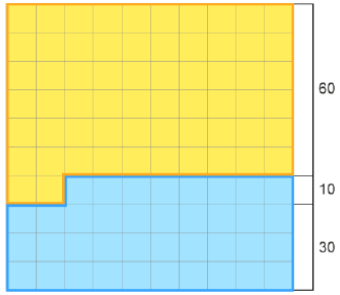
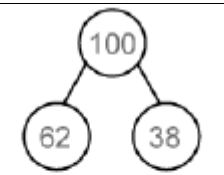
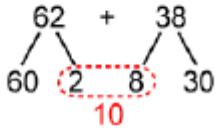
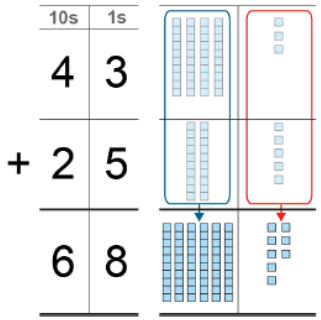
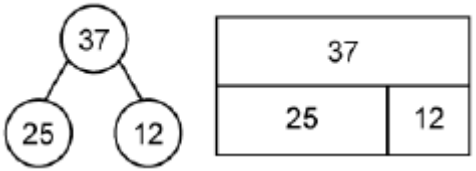
Year 3

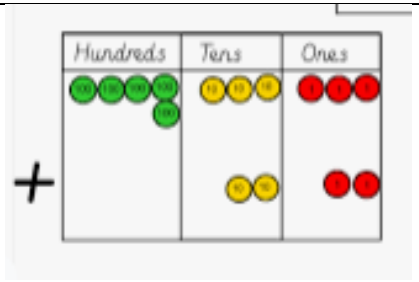
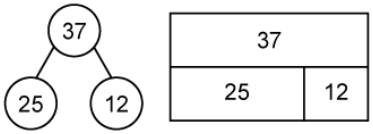
	Core Concrete > pictorial > abstract	Complementary Contexts
3NPV-1 Know that 10 tens are equivalent to 1 hundred, and that 100 is 10 times the size of 10; apply this to identify and work out how many 10s there are in other three- digit multiples of 10.	  **Ten 10-value place-value counters in a tens frame** Know that 100 is 10 times the size of 10 Pupils must then be able to work out how many tens there are in other three-digit multiples of 10.  	

	How many 10cm lengths can a 310cm length of ribbon be cut into?	
3NPV-2 Recognise the place value of each digit in *three*-digit numbers, and compose and decompose *three*-digit numbers using standard and non-standard partitioning.	 Representations of the place-value composition of 342 They must be able to combine units of ones, tens and hundreds to compose three-digit numbers, and partition three-digit numbers into these units. What number is represented by these counters? 1. 	
3NPV-3 Reason about the location of any *three*- digit number in the linear number system, including identifying the previous and next multiple of 100 and 10.	Pupils need to be able to identify or place three-digit numbers on marked number lines with a variety of scales.  Pupils need to be able to identify previous and next multiples of 100 or 10 without the support of a number line.	

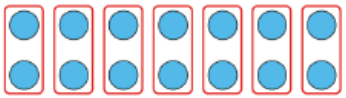
3NPV-4 Divide 100 into 2, 4, 5 and 10 equal parts, and read scales/number lines marked in multiples of 100 with 2, 4, 5 and 10 equal parts.	Pupils should practise counting in multiples of 10, 20, 25, and 50 from 0, or from any multiple of these numbers, both forwards and backwards as these numbers are key to reading scales. Children should be able to count up and down on number number lines. 	Bar models are useful for showing the parts of the whole here. 
3NF-1 Secure fluency in addition and subtraction facts that bridge 10, through continued practice.	This should be a mental method but the children may find the base 10 useful for visualising the key number bonds needed to Recall number facts up to and eyond 10. Mr Kahn drove 8km to get to his friend's house, and then drove another 3km with his friend to get to the gym. How far did Mr Kahn drive?	
3NF-2 Recall multiplication facts, and corresponding division facts, in the 10, 5, 2, 4 and 8 multiplication tables, and recognise products in these multiplication tables as multiples of the	It is important to provide opportunities for pupils to verbalise each multiplication fact as part of the process of developing fluency.  A spider has 8 legs. If there are 5 spiders, how many legs are there altogether?	

corresponding number.		
3NF-3 Apply place-value knowledge to known additive and multiplicative number facts (scaling facts by 10).	Pupils must also be able to combine these facts with unitising in tens, including: • scaling known additive facts within 10, for example, 90 − 60 = 30 • scaling known additive facts that bridge 10, for example, 80 + 60 = 140 • scaling known multiplication tables facts, for example, 30 × 4 = 120 • scaling division facts derived from multiplication tables, for example, 120 ÷ 4 = 30 For calculations such as 80 + 60 = 140 , pupils can begin by using tens frames and counters as they did for calculation across 10, but now using 10-value counters. 	

3AS–1 Calculate complements to 100.	 A shaded 100 grid can be used to show why there are only 9 full tens in the correct complements to 100. The 10th ten is composed of the ones digits. A dressmaker had 1m of ribbon. Then she used 22cm of it. How many centimetres of ribbon does she have left?	 
3AS–2 Add and subtract up to three-digit numbers using columnar methods.	 Columnar addition with no regrouping: calculation and Dienes representation. In Y3 the children also need to be able to complete the addition using place value counters and place value charts. 533 + 22 =	

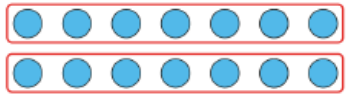
		
3AS–3 Manipulate the additive relationship: Understand the inverse relationship between addition and subtraction, and how both relate to the part-part-whole structure. Understand and use the commutative property of addition, and understand the related property for subtraction.	Pupils must understand that the simplest addition and subtraction equations describe the relationship between 3 numbers, where one is a sum of the other two. They should understand that both addition and subtraction equations can be used to describe the same additive relationship. They should practise writing the full set of 8 equations that are represented by a given partitioning diagram or bar model.  25 + 12 = 37 12 + 25 = 37 37 = 25 + 12 37 = 12 + 25 37 – 12 = 25 37 – 25 = 12 25 = 37 – 12 12 = 37 – 25	
3MD–1 Apply known multiplication and division facts to solve contextual problems with different structures,	Pupils should learn that the commutative property allows them to use their known facts to solve problems about 5, 10, 2, 4 or 8 equal groups (for example, 2 groups of 7).	

including quotitive and partitive division.



7 groups of 2

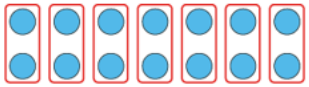
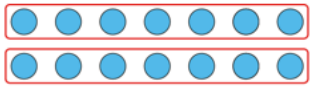
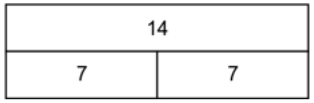
$7 \times 2 = 14$



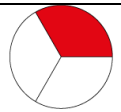
2 groups of 7

$2 \times 7 = 14$

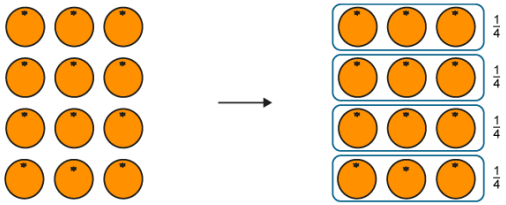
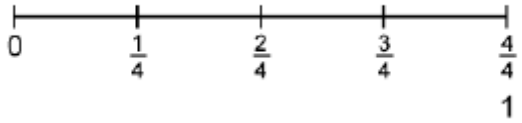
$7 \times 2 = 2 \times 7$

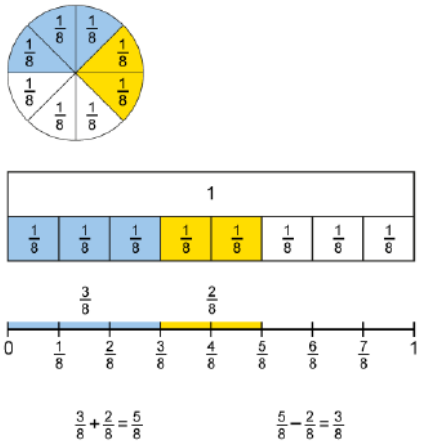
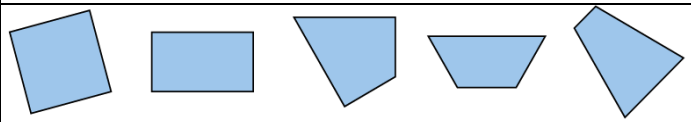
Quotitive division	Partitive division
I need 14 ping-pong balls. There are 2 ping-pong balls in a pack. How many packs do I need?	£14 is shared between 2 children. How much money does each child get?
	
$14 \div 2 = 7$	$14 \div 2 = 7$
	
Figure 27: using an array and bar model to show that 14 divided into groups of 2 is equal to 7	**Figure 28: using an array and bar model to show that 14 shared between 2 is equal to 7**
Language focus "7 times 2 is 14, so 14 divided by 2 is 7." "14 divided into groups of 2 is equal to 7."	**Language focus** "7 times 2 is 14, so 14 divided by 2 is 7." "£14 shared between 2 is equal to £7 each."

3F-1 Interpret and write proper fractions to represent 1 or several parts of a

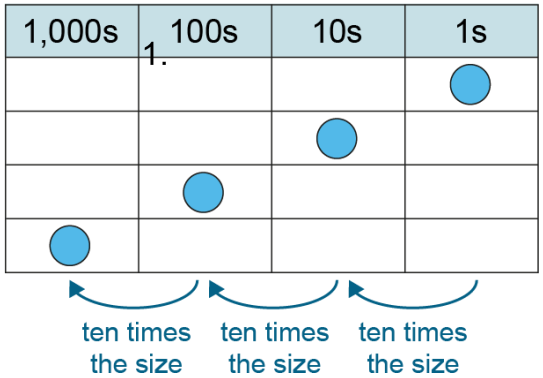
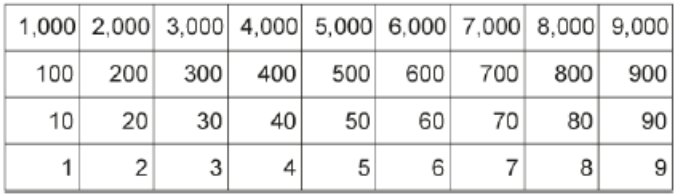
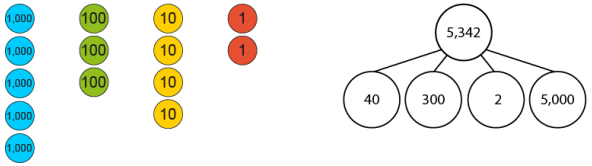


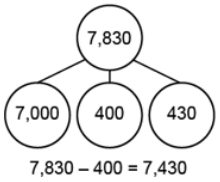
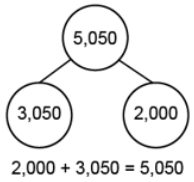
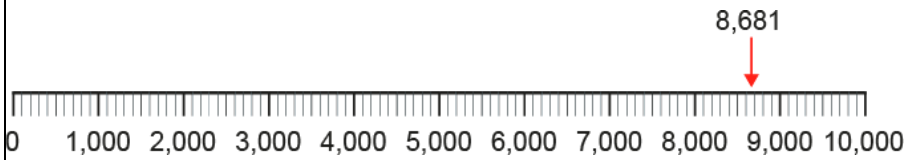
Children should start with fraction tiles and fraction wheels they should learn to identify the whole and the number of equal parts, then to describe one particular equal part relative to the whole.

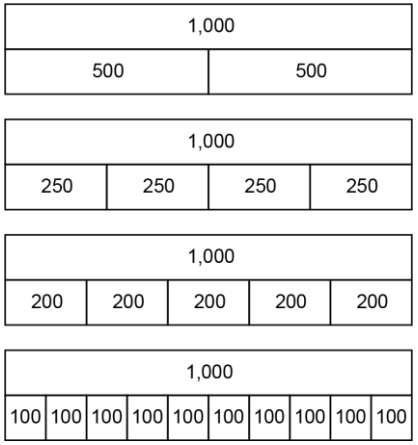
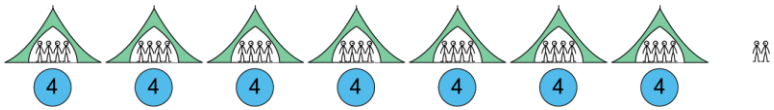
whole that is divided into equal parts.		
3F-2 Find unit fractions of quantities using known division facts (multiplication tables fluency).	Children should identify the whole, the number of equal parts, and the size of each part relative to the whole written as a unit fraction.  To find 1/5 of 15, we divide 15 into 5 equal parts. 15 divided by 5 is equal to 3, so 1/5 of 15 is equal to 3.	
3F-3 Reason about the location of any fraction within 1 in the linear number system.	 Number line to support counting to 1 in multiples of one quarter	
3F-4 Add and subtract fractions with the same denominator, within 1.	To add and subtract fractions, pupils must already understand that non-unit fractions are repeated additions of unit fractions,	

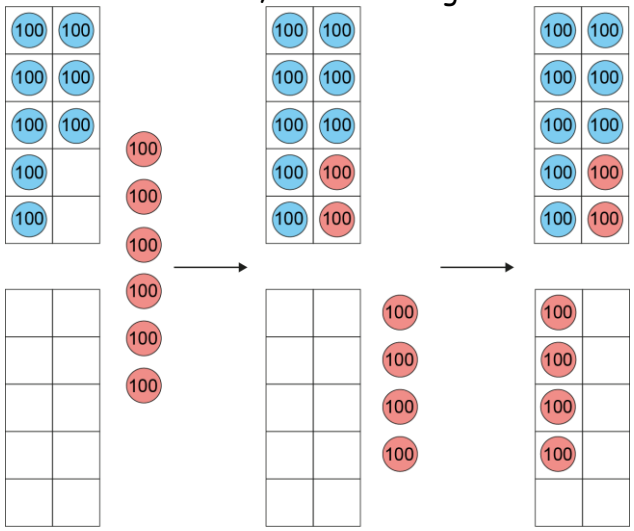
	for example, three-eighths is 3 one-eighths..  	
3G–1 Recognise right angles as a property of shape or a description of a turn, and identify right angles in 2D shapes presented in different orientations.	Pupils should begin by making quarter turns with their bodies, following instructions such as "Stand, and make a quarter turn clockwise. Walk in a straight line. Stop. Make a quarter turn anticlockwise." They should be able to relate these movements to the quarter turn of a clock hand. Pupils should learn that the angle relative to the starting orientation, created by a quarter turn (in either direction), is called a right angle	
3G–2 Draw polygons by joining marked points, and identify parallel and perpendicular sides.	 Look at these 5 quadrilaterals. Mark all the pairs of parallel sides. Hint: you can extend sides to help you.	

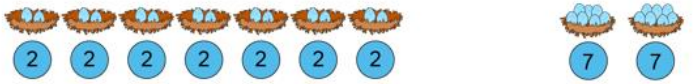
Year 4

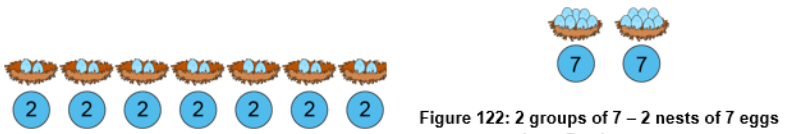
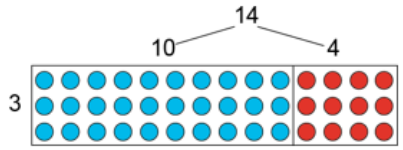
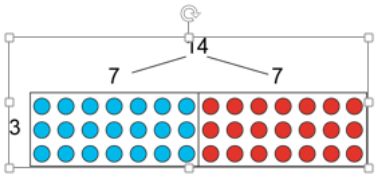
	Core Concrete > pictorial > abstract	Complementary Contexts
4NPV-1 Know that 10 hundreds are equivalent to 1 thousand, and that 1,000 is 10 times the size of 100; apply this to identify and work out how many 100s there are in other four-digit multiples of 100.	 Ten 100-value place-value counters in a tens frame. Pupils must then be able to work out how many hundreds there are in other four-digit multiples of 100.  How many 100g servings of rice are there in a 2,500g bag?	
4NPV-2 Recognise the place value of each digit in *four*-digit numbers, and compose and	2 representations of the place-value composition of 5,342 	

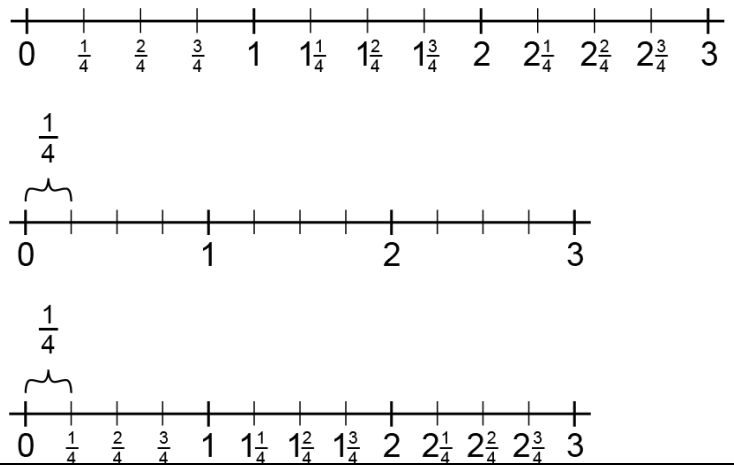
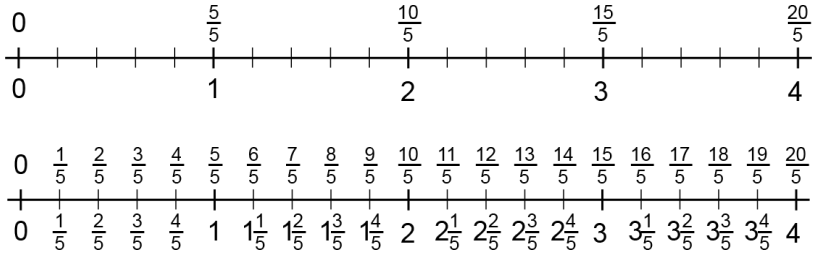
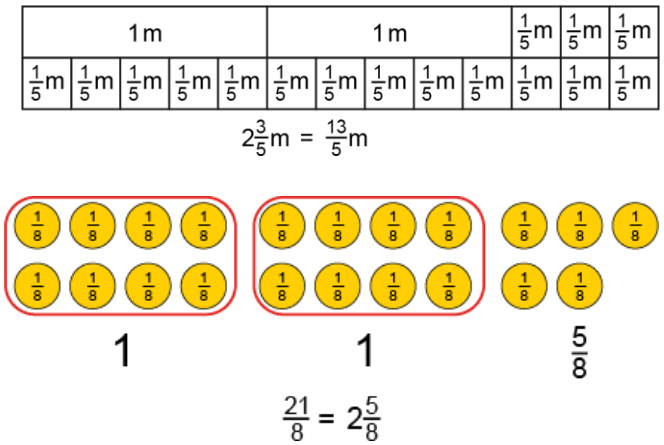
decompose *four*-digit numbers using standard and non-standard partitioning.	Non standard partitioning.  7,830 – 400 = 7,430 Figure 5: partitioning 7,830 into 7,430 and 400  2,000 + 3,050 = 5,050 Figure 6: partitioning 5,050 into 2,000 and 3,050 A football stadium can hold 6,430 people. So far 4,000 tickets have been sold for a match. How many tickets are left?	
4NPV-3 Reason about the location of any *four*-digit number in the linear number system, including identifying the previous and next multiple of 1,000 and 100, and rounding to the nearest of each.	 Using a number line to identify the previous and next multiple of 1,000	

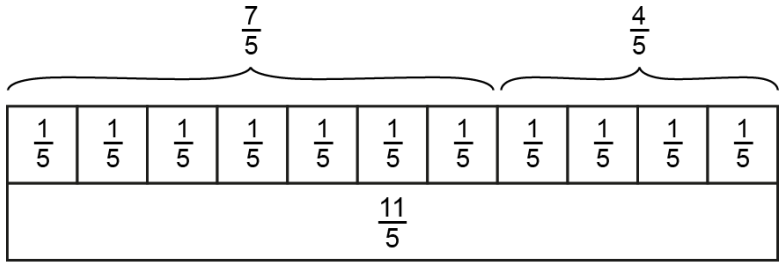
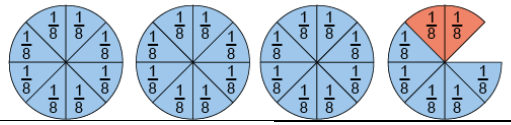
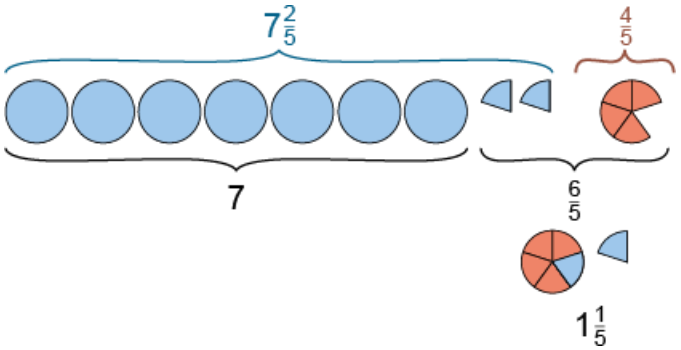
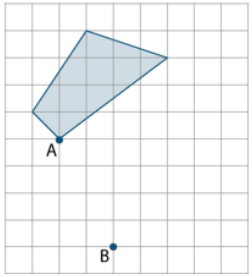
4NPV–4 Divide 1,000 into 2, 4, 5 and 10 equal parts, and read scales/number lines marked in multiples of 1,000 with 2, 4, 5 and 10 equal parts.		
4NF–1 Recall multiplication and division facts up to 12 × 12, and recognise products in multiplication tables as multiples of the corresponding number.		
4NF–2 Solve division problems, with two-digit dividends and one-digit divisors, that involve	Pupils should recognise that a remainder arises when there is something 'left over' in a division calculation. Pupils should recognise and understand why remainders only occur when the dividend is not a multiple of the divisor. This can be achieved by discussing the patterns seen when the dividend is incrementally increased by 1 while the divisor is kept the same.	

remainders, for example: 74 ÷ 9 = 8 r 2 and interpret remainders appropriately according to the context.		
4NF–3 Apply place-value knowledge to known additive and multiplicative number facts (scaling facts by 100), for example: and 14 − 6 = 8 so 800 + 600 = 1,400 1,400 − 600 = 800 3 × 4 = 12 and 12 ÷ 4 = 3 so	For calculations such as 800 + 600 = 1,400, pupils can begin by using tens frames and counters as they did for calculation across 10 and across 100, but now using 100-value counters. 	

4MD–1 Multiply and divide whole numbers by 10 and 100 (keeping to whole number quotients); understand this as equivalent to making a number 10 or 100 times the size.	<table><tr><td>1,000</td><td>2,000</td><td>3,000</td><td>4,000</td><td>5,000</td><td>6,000</td><td>7,000</td><td>8,000</td><td>9,000</td></tr><tr><td>100</td><td>200</td><td>300</td><td>400</td><td>500</td><td>600</td><td>700</td><td>800</td><td>900</td></tr><tr><td>10</td><td>20</td><td>30</td><td>40</td><td>50</td><td>60</td><td>70</td><td>80</td><td>90</td></tr><tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr></table> × 10 ÷ 10 80 × 10 = 800 80 ÷ 10 = 8 The Gattegno chart can be used to help pupils see, for example, that 80, made 10 times the size is 800: pupils can move their finger or a counter *up* from 80 to 800. They should connect this action to multiplication by 10, and be able to solve/write the corresponding multiplication calculation (80 □10 □ 800). Similarly, because 80 is 10 times the size of 8, they can solve 80 □10 □ 8 , moving their finger or a counter *down* from 80 to 8. ×100 <table><tr><th>1,000s</th><th>100s</th><th>10s</th><th>1s</th></tr><tr><td></td><td></td><td></td><td>8</td></tr><tr><td></td><td>8</td><td>0</td><td>0</td></tr></table> ÷ 100 8 × 100 = 800 800 ÷ 100 = 8	
4MD–2 Manipulate multiplication and division equations, and understand and apply the commutative property of multiplication.	 7 groups of 2 7 × 2 = 14 2 groups of 7 2 × 7 = 14 7 × 2 = 2 × 7	7 × 2 = 14 2 × 7 = 14 Interpretation 1 Interpretation 2 

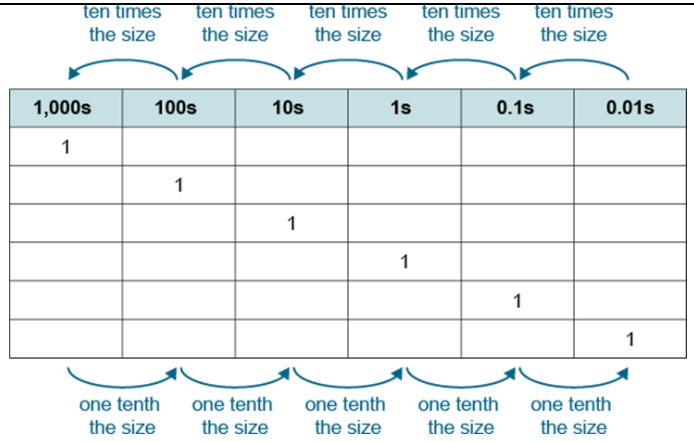
		$14 \div 7 = 2$ **Partitive division** **Quotitive division**  **Figure 122: 2 groups of 7 – 2 nests of 7 eggs and two 7-value counters**
4MD-3 Understand and apply the distributive property of multiplication.	Pupils should then use the distributive property and known multiplication table facts to multiply 2-digit numbers (above 12) by one-digit numbers. 14 x 3 , for example, can be calculated by relating it to 10 x 3 and 4 x 3 :  $14 \times 3 = 10 \times 3 + 4 \times 3$ $= 30 + 12$ $= 42$  $14 \times 3 = 7 \times 3 + 7 \times 3$ $= 21 + 21$ $= 42$	

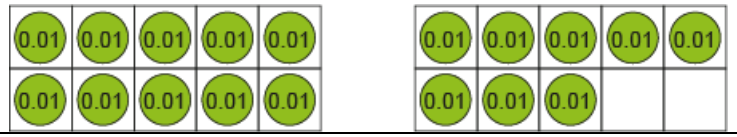
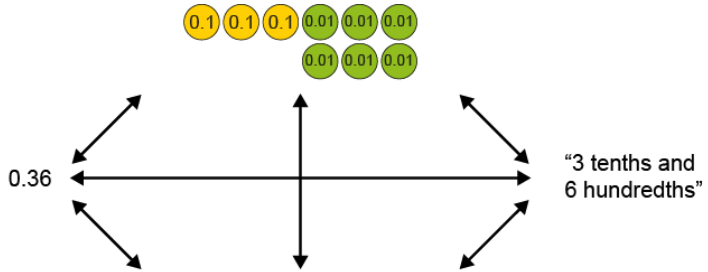
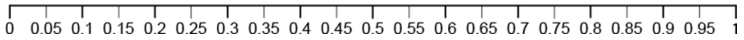
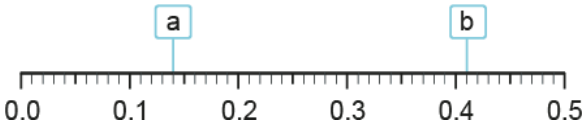
4F-1 Reason about the location of mixed numbers in the linear number system.		
4F-2 Convert mixed numbers to improper fractions and vice versa		

4F–3 Add and subtract improper and mixed fractions with the same denominator, including bridging whole numbers, for example: $\frac{7}{5}+\frac{4}{5}=\frac{11}{5}$ $3\frac{7}{8}-\frac{2}{8}=3\frac{5}{8}$	 $\frac{7}{5}+\frac{4}{5}=\frac{11}{5}$  $3\frac{7}{8}-\frac{2}{8}=3\frac{5}{8}$ 	
4G–1 Draw polygons, specified by coordinates in the first quadrant, and translate within the first quadrant.	Translate the quadrilateral so that point A moves to point B. 	
4G–2 Identify regular polygons, including equilateral triangles and squares, as those in which the side-		

lengths are equal and the angles are equal. Find the perimeter of regular and irregular polygons.		
4G–3 Identify line symmetry in 2D shapes presented in different orientations. Reflect shapes in a line of symmetry and complete a symmetric figure or pattern with respect to a specified line of symmetry.		

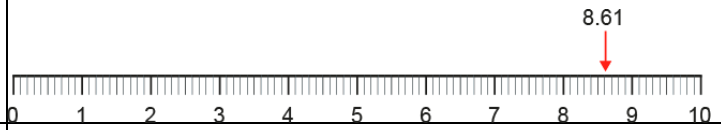
Year 5

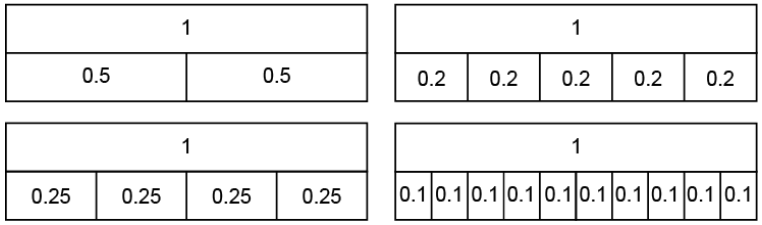
	Core Concrete > pictorial > abstract	Complementary Contexts
5NPV-1 Know that 10 tenths are equivalent to 1 one, and that 1 is 10 times the size of 0.1. Know that 100 hundredths are equivalent to 1 one, and that 1 is 100 times the size of 0.01. Know that 10 hundredths are equivalent to 1 tenth, and that 0.1 is 10 times the size of 0.01.	 The value of a given digit is made 10 times the size if it is moved 1 position to left, and is made one tenth times the size if it is moved 1 position to the right. Pupils should learn, therefore, that we can extend the place-value chart to include positions to the right of the ones place. • 18 hundredths is equal to 1 tenth and 8 hundredths, and is written as 0.18 • 18 tenths is equal to 1 one and 8 tenths, and is written as 1.8	 The number 300 is spoken as "three hundred" rather than as "three-zero-zero", and this helps pupils to identify the value of the 3 in 300. However, decimal fractions are usually spoken as digits, for example, 0.03 is spoken as "zero-point-zero-three" (or "nought- point-nought-three") rather as "three hundredths". As such, pupils need to practise speaking decimal fractions in both ways and learn to convert from one to the other.

		
5NPV-2 Recognise the place value of each digit in numbers with up to 2 decimal places, and compose and decompose numbers with up to 2 decimal places using standard and non-standard partitioning.	**2 representations of the place-value composition of 53.42**  1,000 2,000 3,000 4,000 5,000 6,000 7,000 8,000 9,000 100 200 300 400 500 600 700 800 900 10 20 30 40 50 60 70 80 90 1 2 3 4 5 6 7 8 9 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 0.01 0.02 0.03 0.04 0.05 0.06 0.07 0.08 0.09	
5NPV-3 Reason about the location of any number with up to 2 decimals places in the linear number system, including identifying the previous and next multiple of 1 and	  Pupils need to become familiar with the relative positons, on a number line, of numbers with 1 and 2 decimal places. They will need to see number lines with both tenths and intermediate	

<table>
<tr>
<td>0.1 and rounding to the nearest of each.</td>
<td>hundredths values marked, and learn, for example, that 0.5 is the same as

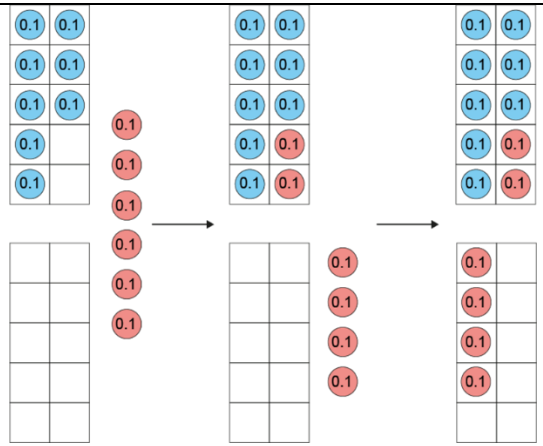
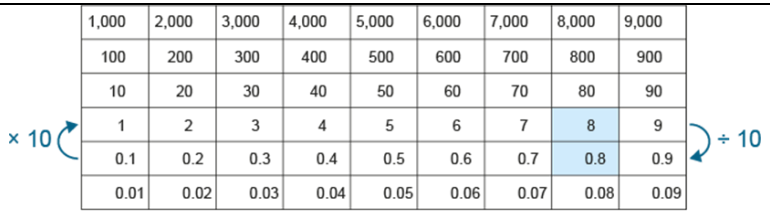
0.50 and 3 is the same as 3.0 or 3.00.

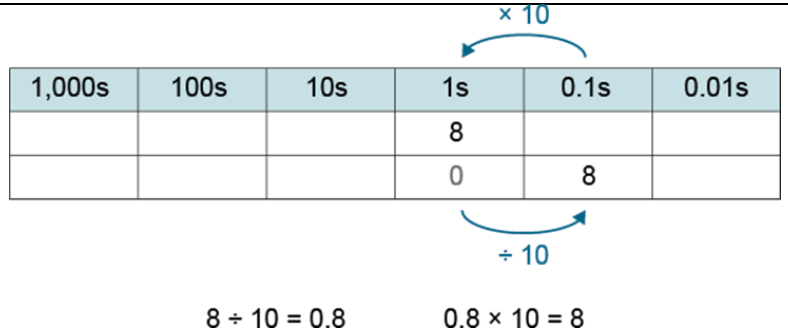
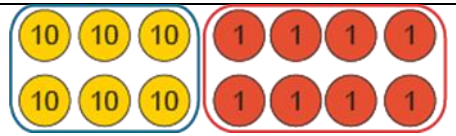
 
</td>
<td></td>
</tr>
<tr>
<td>5NPV-4 Divide 1 into 2, 4,

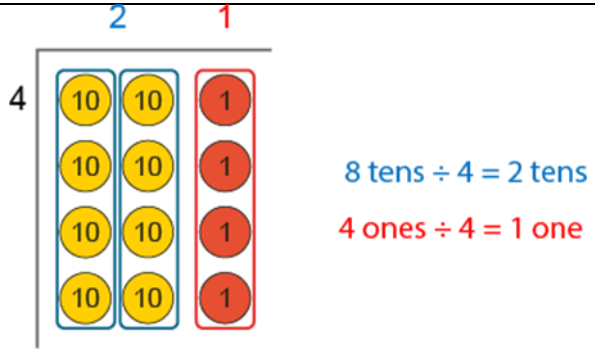
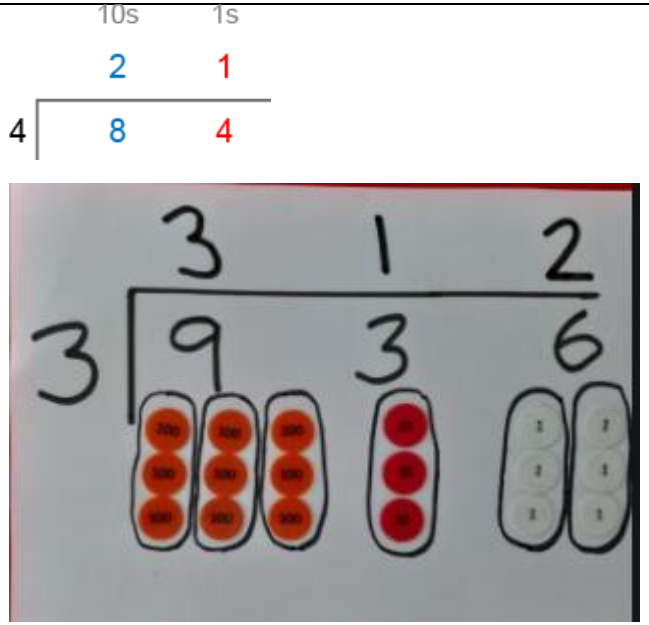
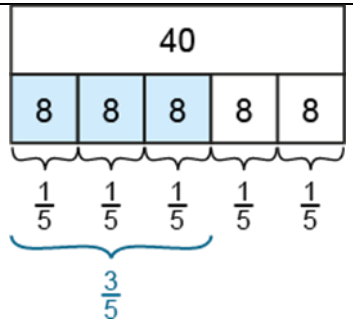
5 and 10 equal parts, and read scales/number lines marked in units of 1 with 2, 4, 5 and 10 equal parts.</td>
<td>Pupils should practise counting in multiples of 0.1, 0.2, 0.25 and 0.5 from 0, or from any multiple of these numbers, both forwards and backwards. This is an important step in becoming fluent with these number patterns.</td>
<td> 
</td>
</tr>
<tr>
<td>5NPV-5 Convert between units of measure, including using common decimals and fractions.</td>
<td>Practical experience of these conversions will help pupils to avoid common errors in recalling the correct power of 10 for a given conversion. For example, they can walk 1km while counting the number of metres using a trundle measuring wheel.

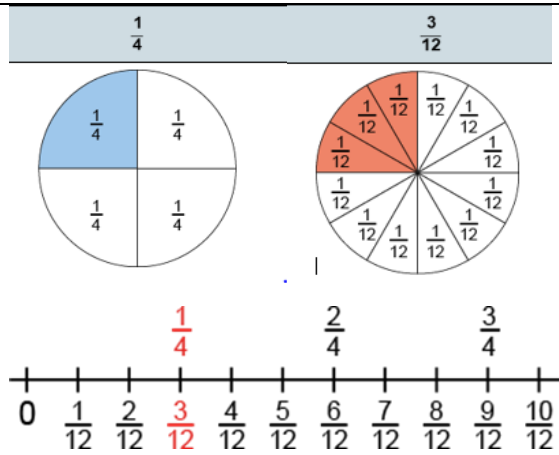
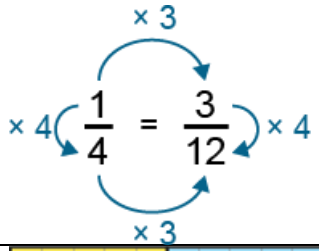
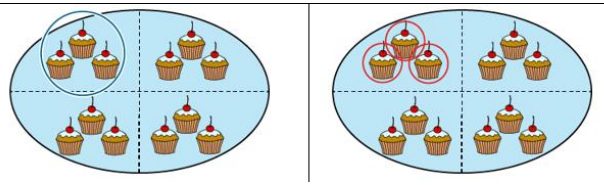
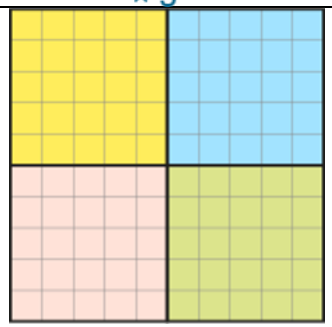
Pupils can use ratio tables for support.

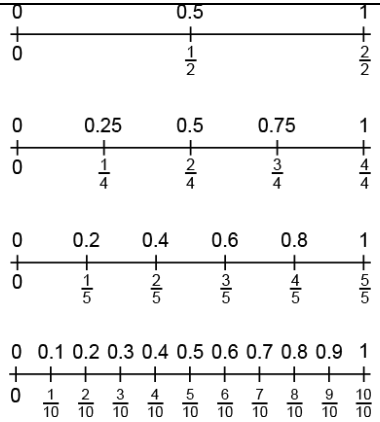
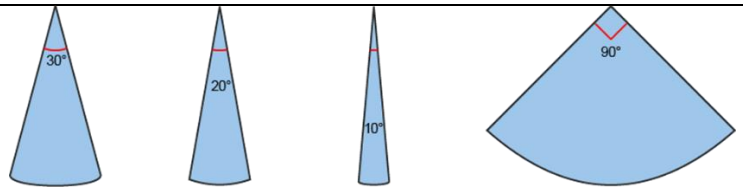
<table>
<tr><td>1m</td><td>100cm</td></tr>
<tr><td>$\frac{3}{4}$m</td><td>75cm</td></tr>
</table>
<table>
<tr><td>1,000ml</td><td>1 litre</td></tr>
<tr><td>3,700ml</td><td>3.7 litres</td></tr>
</table>
<table>
<tr><td>100p</td><td>£1</td></tr>
<tr><td>52p</td><td>£0.52</td></tr>
</table></td>
<td></td>
</tr>
</table>

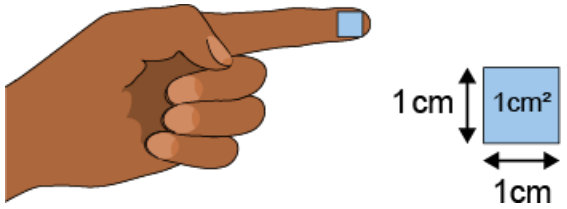
5NF–1 Secure fluency in multiplication table facts, and corresponding division facts, through continued practice.	Pupils must also be able to fluently derive related division facts, including division facts with remainders before they begin to learn formal written methods for multiplication and division	
5NF–2 Apply place-value knowledge to known additive and multiplicative number facts (scaling factsby 1 tenth or 1 hundredth),for example: 8 + 6 = 14 0.8 + 0.6 = 1.4 0.08 + 0.06 = 0.14 3 × 4 = 12 0.3 × 4 = 1.2 0.03 × 4 = 0.12	 Tens frames with 0.1-value counters showing 0.8 + 0.6 = 1.4	
5MD–1 Multiply and divide numbers by 10 and 100; understand this as equivalent to making a number 10 or 100 times the size, or 1	 8 ÷ 10 = 0.8 0.8 × 10 = 8	

tenth or 1 hundredth times the size.	 8 ÷ 10 = 0.8 0.8 × 10 = 8	
5MD–2 Find factors and multiples of positive whole numbers, including common factors and common multiples, and express a given number as a product of 2 or 3 factors	Children should learn the definitions of the terms 'multiple' and 'factor', and understand the inverse relationship between them. Pupils should learn to express multiples of 10 or 100 as products of 3 factors, for example: 7 x 3 = 21 so 7 x 3 x 10 = 210	
5MD–3 Multiply any whole number with up to 4 digits by any one-digit number using a formal written method.	 34 x 2 = Place-value counters showing 34 x 2	$34 \times 2 = 30 \times 2 + 4 \times 2$ $= 60 + 8$ $= 68$ 10s 1s 3 4 × 2 6 8

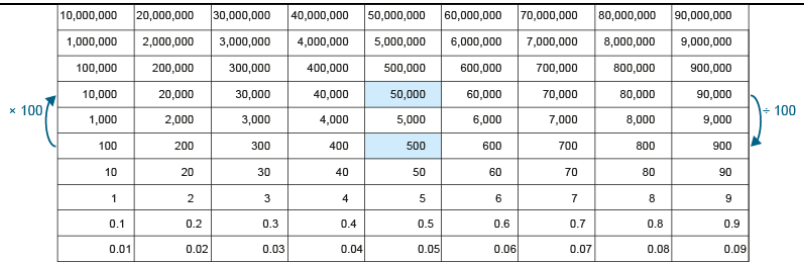
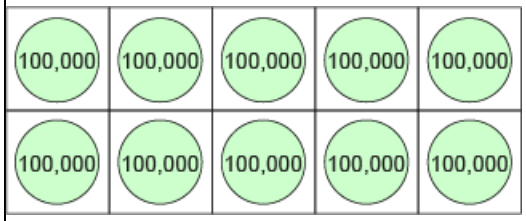
5MD-4 Divide a number with up to 4 digits by a one-digit number using a formal written method, and interpret remainders appropriately for the context.	 8 tens ÷ 4 = 2 tens 4 ones ÷ 4 = 1 one Short division using place value counters 1 5 3 4 6 2 1 1 2	
5F-1 Find non-unit fractions of quantities.	 Three-fifths is equal to 3 one-fifths. To find 3 one-fifths of 40, first find one-fifth of 40 by dividing by 5, and then multiply by 3.	

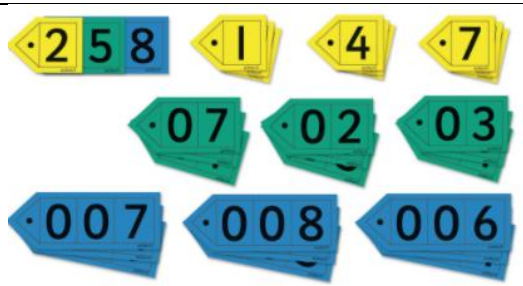
5F-2 Find equivalent fractions and understand that they have the same value and the same position in the linear number system.	 	
5F-3 Recall decimal fraction equivalents for $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{1}{10}$, and for multiples of these proper fractions.	 A coloured number square is useful here. Hundred grid divided into 4 equal parts: $\frac{1}{4}$ is equal to 25 hundredths	

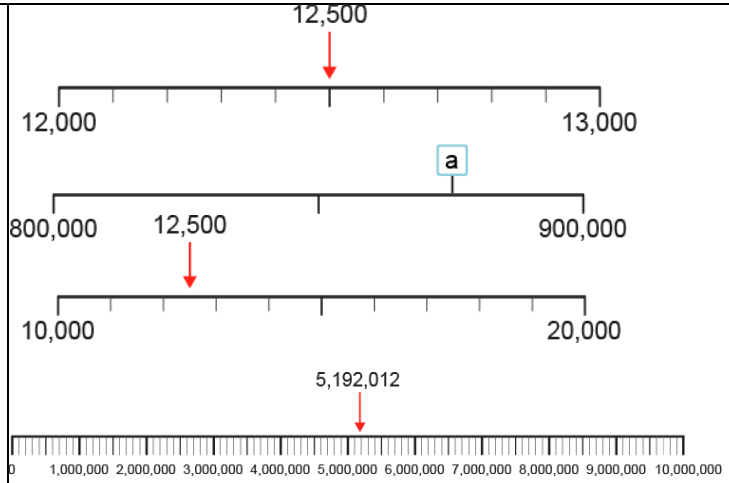
	 0 to 1 number lines illustrating common proper fraction – decimal fraction equivalents	
5G–1 Compare angles, estimate and measure angles in degrees (°) and draw angles of a given size.		

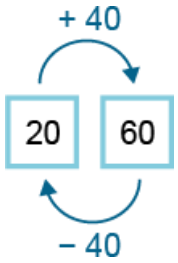
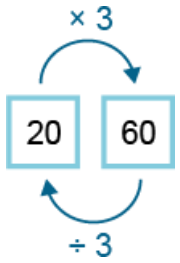
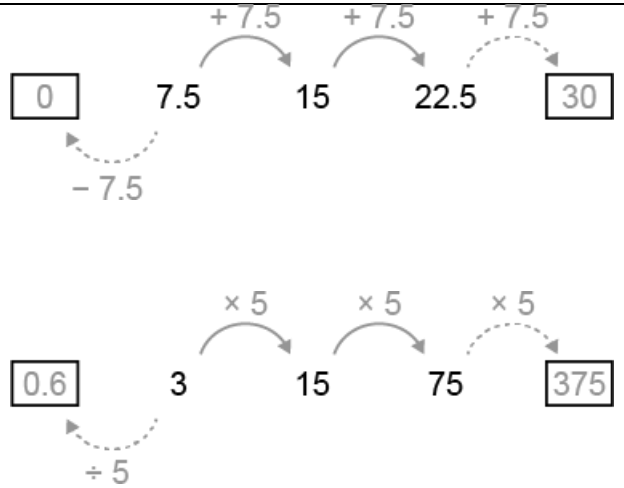
5G–2 Compare areas and calculate the area of rectangles (including squares) using standard units.	Pupils should learn that, when there is not a clear visual difference between areas, a common unit can be used to quantify the areas and enable comparison. They should understand that any unit can be used, but that the square centimetre (cm^2) is the standard unit of measure for area that they will use most frequently. Pupils should gain a sense of the size of a square centimetre, and the notation used, before they begin to quantify other areas using this unit.  a square centimetre	

Year 6

	Core Concrete > pictorial > abstract	Complementary Contexts
6NPV–1 Understand the relationship between powers of 10 from 1 hundredth to 10 million, and use this to make a given number 10, 100, 1,000, 1 tenth, 1 hundredth or 1 thousandth times the size (multiply and divideby 10, 100 and 1,000).	 10,000,000 20,000,000 30,000,000 40,000,000 50,000,000 60,000,000 70,000,000 80,000,000 90,000,000 1,000,000 2,000,000 3,000,000 4,000,000 5,000,000 6,000,000 7,000,000 8,000,000 9,000,000 100,000 200,000 300,000 400,000 500,000 600,000 700,000 800,000 900,000 10,000 20,000 30,000 40,000 50,000 60,000 70,000 80,000 90,000 1,000 2,000 3,000 4,000 5,000 6,000 7,000 8,000 9,000 100 200 300 400 500 600 700 800 900 10 20 30 40 50 60 70 80 90 1 2 3 4 5 6 7 8 9 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 0.01 0.02 0.03 0.04 0.05 0.06 0.07 0.08 0.09 Pupils should know that each power of 10 is equal to 1 group of 10 of the next smallest power of 10, for example 1 million is equal to 10 hundred thousands.  Ten 100,000-value place-value counters in a tens frame Complete the sentences. 1. a. 500 made 1,000 times the size is_. b. 0.7 made 100 times the size is___. c. 800,000 made 10 times the size is_____. d. 4,000,000 made one-thousandth times the size is____. e. 9,000 made one-hundredth times the size is___.	

	f. 3 made one-tenth times the size is_____.	
6NPV–2 Recognise the place value of each digit in numbers up to 10 million, including decimal fractions, and compose and decompose numbers upto 10 million using standard and non-standard partitioning.	 Pupils can initially use place-value counters for support with this type of partitioning 	
6NPV–3 Reason about the location of any number up to 10 million, including decimal fractions, in the linear number system, and round numbers, as appropriate,	Pupils need to be able to estimate the value or position of numbers on unmarked or partially marked numbers lines, using appropriate proportional reasoning.	

including in contexts.		
6NPV–4 Divide powers of 10, from 1 hundredth to 10 million, into 2, 4, 5and 10 equal parts, andread scales/number lines with labelled intervals divided into 2, 4, 5 and 10 equal parts.	It is important for pupils to be able to divide powers of 10 into 2, 4, 5 or 10 equal parts because these are the intervals commonly found on measuring instruments and graph scales. Pupils have already learnt to divide 1, 100 and 1,000 in this way, and must now extend this to larger powers of 10. Pupils should be able to make connections between powers of 10, for example, describing similarities and differences between the values of the parts when 1 million, 1,000 and 1 are divided into 4 equal parts. 1,000,000 250,000 | 250,000 | 250,000 | 250,000 1,000 250 | 250 | 250 | 250 1 0.25 | 0.25 | 0.25 | 0.25	

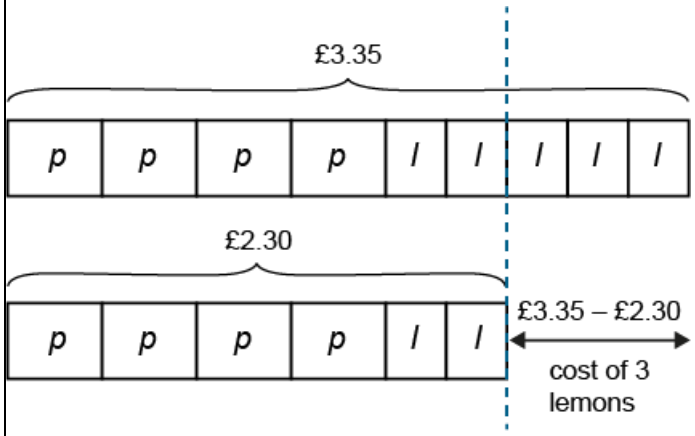
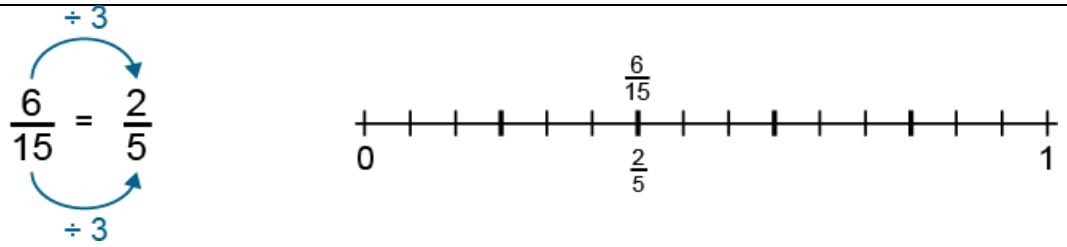
6AS/MD–1 Understandthat 2 numbers can be related additively or multiplicatively, and quantify additive and multiplicative relationships (multiplicative relationships restricted to multiplication by a whole number).	additive relationship  multiplicative relationship 	
6AS/MD–1 Use a given additive or multiplicative calculation to derive or complete a related calculation, using arithmetic properties, inverse relationships, and place-	If one addend is increased and the other is decreased by the same amount, the sum stays the same. Pupils should be able to use the compensation property of addition to complete equations such as 25 + 35 = 27.5 + ? , and to help them solve calculations such as 27.5 + 32.5. If I multiply one factor by a number, I must divide the other factor by the same number for the product to stay the same.	

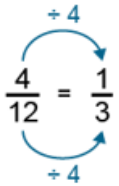
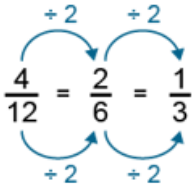
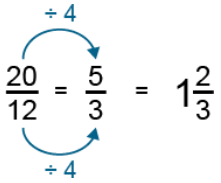
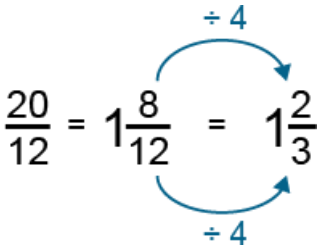
<table>
<tr><td>value understanding.</td><td></td><td></td></tr>
<tr><td><u>6AS/MD–3</u>
Solve problems involving ratio relationships.</td><td>

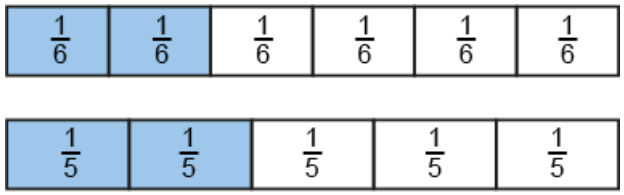
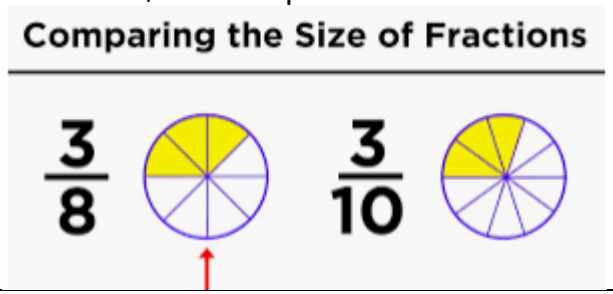

<table>
<tr><td>number of red beads</td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>number of blue beads</td><td>3</td><td>6</td><td>9</td><td>12</td></tr>
<tr><td>total number of beads</td><td>4</td><td>8</td><td>12</td><td>16</td></tr>
</table>
Ratio table

Pupils should also be able to answer questions such as:

• if there were 5 red beads, how many blue beads would there be?
• if there were 21 blue beads, how many beads would there be altogether?
• if there were 40 beads altogether, how many red beads and how many blue beads would there be?
</td><td></td></tr>
</table>

6AS/MD–4 Solveproblems with 2 unknowns.	Bar model - Pupils should also be able to use bar modelling to solve more complex problems with 2 unknowns and 1 solution, such as: 4 pears and 5 lemons cost £3.35. 4 pears and 2 lemons cost £2.30. What is the cost of 1 lemon?  cost of 3 lemons = £3.35 – £2.30 = £1.05 so cost of 1 lemon = £1.05 ÷ 3 = £0.35	
6F–1 Recognise whenfractions can be simplified, and use common factors to simplify fractions.	 Simplifying 6/15 by dividing the numerator and denominator by the common factor of 3	

	Pupils should then learn to simplify fractions where the numerator and denominator share several common factors. Pupils should understand that they should divide the numerator and denominator by the highest common factor to express a fraction in its simplest form, but that the simplification can also be performed in more than 1 step.  Figure 21: simplifying $\frac{4}{12}$ by dividing the numerator and denominator by the highest common factor  Figure 22: simplifying $\frac{4}{12}$ in 2 steps   	

6F–2 Express fractions in a common denomination and use this to compare fractions that are similar in value.	$\frac{1}{3} = \frac{8}{24}$ (×8) $\qquad \frac{3}{8} = \frac{9}{24}$ (×3) Children can use fraction wheels to visualise the comparison before moving onto the method above.	
6F–3 Compare fractions with different denominators, including fractions greater than 1, using reasoning, and choose between reasoning and common denomination as a comparison strategy.	If the numerators are both 1, then the larger the denominator, the smaller the fraction.  $\frac{2}{5} > \frac{2}{6}$ Pupils should now extend this to compare other fractions with the same numerator, for example see above  	

6G–1 Draw, compose, and decompose shapes according to given properties, including dimensions, angles and area, and solve related problems.		